If Approximating Nonlinear Areas, then Consider Fuzzy Systems

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THE FUZZY SYSTEM IS A KNOWLedge-based system consisting of linguistic if-then rules. The rules can be constructed using either an expert knowledge of the system that we want to model or system data acquired through experimentation. One of the very attractive aspects of using fuzzy systems is the realization of nonlinear mappings. When one wishes to approximate a nonlinear area and wants to combine a highly transparent representation with a linguistic interpretation in the form of rules, fuzzy systems should be the choice because, in such cases, they are superior over the other nonlinear approximation techniques. The fuzzy-system family classifies mainly upon the structure of the antecedent (if) and consequent (then) part of the rules, and it can be roughly divided into

• *linguistic fuzzy models,* where both parts consist of linguistic variables and linguistic values

• *fuzzy relational models,* where the mapping from the antecedent fuzzy sets A_i to the consequent fuzzy sets B_i is represented by a fuzzy relation

• *Takagi-Sugeno (TS) models,* where the rule antecedents describe fuzzy regions in the input space (as with the linguistic models) and the rule consequents are crisp functions of the model inputs, i.e., **if** x is A_i **then** $y_i = f_i(x)$.

This article will focus only on the TS-type models. More on fuzzy modeling can be found in the paper by Babuška and Verbrüggen.

Consider now a nonlinear area created by the outputs of a family of functions that we use to describe a system with uncertain parameters. The main question is how to provide a mathematical representation of the system on the basis of the nonlinear area? Fuzzy systems seem a very sound choice; in fact, Škrjanc et al. introduced a method where by fixing the rule antecedents (membership functions) and establishing the rule consequents such that when the parameters of the consequent part vary in a certain interval, one is able to define the upper and lower boundaries of the area. The so-called interval fuzzy model (INFUMO) is in this case very appealing for two reasons: first, the optimization of the INFUMO parameters is based on linear programming and is therefore easy to implement and not computationally demanding; second, in providing the area boundaries not only do we get the area approximation but also the confidence band in which all possible system outputs can be found with the probability value of one. This is particularly important when one seeks a robust description of an uncertain system that also comprises the effects of the unknown system's inputs in its nominal operating mode. Hence, if the system's output crosses the boundaries, it can be concluded that, knowing that the uncertainties are already covered by the boundaries, a fault has occurred in the system.

In this study, we are dealing with a system whose parameters vary in a certain tolerance band. The band is fixed yet unknown to the designer. The problem we are investigating is how to design a simple and efficient fault-detection system or how to make a decision system that will differentiate between an effect of the system uncertainty and a serious fault in the system's operation using only the process input and output data. The issue of how to design a robust fault-detection system has been extensively studied over the past two decades. The majority of papers focus on the construction of the so-called residual generator, a comparator of the process and the process-model output that creates a residual signal. Generally, the signal should be below a preassigned threshold in a nonfault process operation and above the threshold otherwise. The threshold

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is usually connected to the influence of disturbances and other unknown process inputs. Therefore, a great number of papers have been dedicated to the challenge of constructing a robust residual generator that will be insensitive to the influences of disturbances and model uncertainties. Some exciting results have been achieved using various artificial-intelligence techniques such as nonlinear adaptive observers, fuzzy thresholds and fuzzy models, and neural-network-based models. For more information, a dedicated reader is referred to survey papers by by Frank and Ding, Frank and Köppen-Seliger or more specialized papers (Patton and Chen; Zhang et al.).

However, all of these methods have something in common: that the transfer functions from faults and unknown inputs to the output must be known to draw a distinction between a fault and the influence of unknown inputs. Since we are dealing with a class of interval-type uncertain systems where the influence of the unknown inputs cannot be modeled and only the input-output data are available, the methods in question are not suitable. The idea we propose is to transform the data, corresponding to the nominal system operation including disturbances, into a nonlinear input-output area, approximate it by the means of the INFUMO and use the INFUMO output as an online residual generator. Figure 1 presents this concept.

By calculating the normalized distance of the transformed system's output to the boundaries provided by the INFUMO, we get a simple and effective fault-detection (FD) system, which is robust to the effects of the unknown inputs. Strong points of the proposed procedure are also intuitiveness, simplicity of design, and the possibility of using sets of piece-wise constant signals of arbitrary amplitude time courses in obtaining the input-output data for identification of the INFUMO.

The effectiveness of our proposed method was tested on a small-scale industrial motor-generator plant. By using the method, a simple solution for real-time detection of load change in the plant was developed. Optimization convergence problems that could have arisen either from too many parameters or from a vast amount of data were solved by implementing a simple data-reduction method. Low-pass filtering was used for the process-data transformation.

Derivation of the INFUMO

The derivation of the INFUMO can be roughly divided into the following stages: applying a fuzzy model in the TS form, interval identification using l_∞-norm, and obtaining the INFUMO parameters using linear programming. A short description of all the stages will be given next. A more detailed insight can be found in Škrjanc et al.

A static fuzzy TS-type model in affine form can be given as a set of rules

$$\mathbf{R}_j : \textbf{if } x_p \textbf{ is } \mathbf{A}_j, \quad \textbf{then } y = \theta_j^T \mathbf{x}, \; j = 1, \ldots, m. \tag{1}$$

The variable x_p denotes the input or variable in premise, and variable y is the output of the model. The antecedent variable is connected with s fuzzy sets $\mathbf{A}_j$, and each fuzzy set $\mathbf{A}_j$ ($j = 1, \ldots, m$) is associated with a real-valued function $\mu_{A_j}(x_p) : R \to [0, 1]$, that produces a membership grade of the variable x_p with respect to the fuzzy set $\mathbf{A}_j$. The consequent vector is denoted $\mathbf{x}^T = [x, 1]$. As the output functions are in affine form, 1 was added to the vector $\mathbf{x}$. The system output is a linear combination of the consequent states, and θ_j is a vector of fuzzy parameters. The system in (1) can be described in closed form

$$y = \beta^T(x_p)\Theta\mathbf{x}, \tag{2}$$

where $\Theta^T = [\theta_1, \ldots, \theta_m]$ denotes a coefficient matrix for the complete set of rules, and $\beta^T(x_p) = [\beta_1(x_p), \ldots, \beta_m(x_p)]$ is a vector of normalized membership functions with elements that indicate the degree of fulfillment of the respective rule. Functions $\beta_j(x_p)$ can be defined as

$$\beta_j(x_p) = \frac{\mu_{A_j}(x_p)}{\sum_{j=1}^{m} \mu_{A_j}(x_p)} \quad j = 1, \ldots, m, \tag{3}$$

and can be seen as a weighted average of the rule contributions to the system output y.

The model parameters are estimated using the l_∞-norm as a criterion for the measure of the modeling error. We will assume that our data can be found in a bounded space and that there exists a family of functions in the space that form a nonlinear area we want to approximate. The idea is to approximate the area, produced by the functions, by its upper and lower boundaries. However, obtaining the exact boundaries would require an infinite amount of data. Since in this case, we are limited to the finite set of measured output values $\mathbf{Y} = \{y_1, y_2, \ldots, y_N\}$ and the finite set of input data $\mathbf{Z} = \{z_1, z_2, \ldots, z_N\}$, the upper and the lower boundary functions will be approximated by fuzzy functions in the form given in (2). The existence condition for the approximation is given by the Stone-Weierstrass theorem (Ying and Chen). According to the theorem,

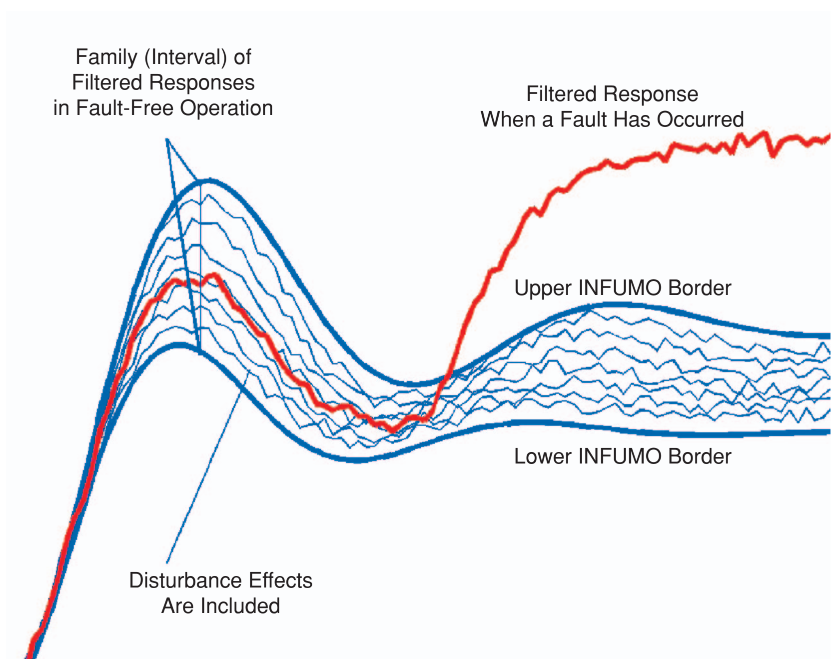

Fig. 1 Fault-detection system using a static INFUMO model

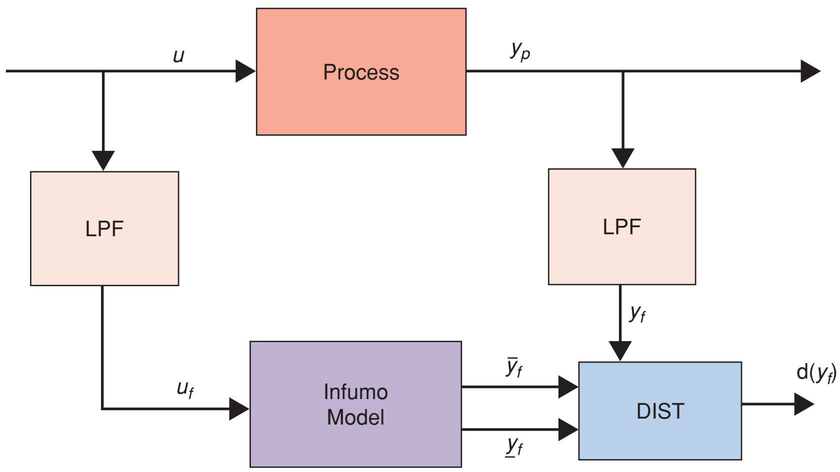

Fig. 2 Fault-detection system using a static INFUMO model

The solutions to both problems can be found by linear programming because both problems can be viewed as solving the min-max optimization problem, as described in detail in the Škrjanc et al. paper. This brings simplicity to the realization of the optimizing process. However, large data sets and a large number of parameters will still pose a threat to optimization convergence. In the first case, we approach the problem with data-reduction methods. In the latter case, we have to find solutions to reduce the number of parameters.

there exists a fuzzy system f such that

$$\max_{z_i \in \mathbf{Z}} \left| f(z_i) - g(z_i) \right| < \varepsilon, \quad \forall i, \qquad (4)$$

i.e., a fuzzy function f can approximate an arbitrary nonlinear function g with any desired degree of accuracy for any ε.

To estimate the optimal parameters of the proposed fuzzy function, two tasks must be accomplished: defining and arranging the membership functions and calculating the optimal fuzzy parameters. Finding the optimal arrangement of membership functions surpasses the scope of this article. Hence, let us assume that linear membership functions are used, and the apex positions are calculated using one of the clustering methods, e.g., c-means clustering (Ying and Chen).

After arranging the membership functions, the optimal fuzzy parameters are obtained by minimization of the maximum modeling error

$$\max_{z_i \in \mathbf{Z}} \left| y_i - f(z_i) \right| = \max_{z_i \in \mathbf{Z}} \left| y_i - \beta^T(x_p)\Theta \mathbf{x}(z_i) \right| \qquad (5)$$

over the whole input set $\mathbf{Z}$. This implies the *min-max* optimization method, and l_∞-norm is used here as the maximum difference between the elements of two vectors. Note that the data are obtained by sampling different output functions y with arbitrary values of z. The idea of robust interval fuzzy modeling can be seen as finding a lower fuzzy function $\underline{f}$ and an upper fuzzy function $\overline{f}$ that satisfy the following condition:

$$\underline{f}(z_i) \le y_i \le \overline{f}(z_i), \quad \forall z_i \in \mathbf{Z}. \qquad (6)$$

This also means that we get a band of data in which all y_i can be found with a value probability of one. The main requirement when defining the band is that it is as narrow as possible within the proposed constraints.

The upper and the lower fuzzy functions, respectively, can be found by solving the following optimization problems for $\forall i$

$$\begin{aligned} &\min_{\underline{\Theta}} \max_{z_i \in \mathbf{Z}} \left| y_i - \beta^T(\mathbf{x}_{pi})\underline{\Theta}\mathbf{x}(z_i) \right|, \\ &\quad \text{if } y_i - \beta^T(\mathbf{x}_{pi})\underline{\Theta}\mathbf{x}(z_i) \ge 0, \\ &\min_{\overline{\Theta}} \max_{z_i \in \mathbf{Z}} \left| y_i - \beta^T(\mathbf{x}_{pi})\overline{\Theta}\mathbf{x}(z_i) \right|, \\ &\quad \text{if } y_i - \beta^T(\mathbf{x}_{pi})\overline{\Theta}\mathbf{x}(z_i) \le 0. \end{aligned} \qquad (7)$$

Residual formation and diagnostic scenario

As was shown by Frank and Ding, all residual generators can be designed by

$$r(t) = Q(p)(y(t) - \tilde{y}(t)), \qquad (8)$$

with $\tilde{y}(t)$ as an output estimation and $Q(p)$ is a filter that is free to design and enhances the residual robustness to the unknown process inputs. Combining (8) with (2), the following relation can be written

$$\begin{aligned} r(t) &= Q(p)(y(t) - \beta^T\Theta\mathbf{u}(t)) \\ &= Q(p)y(t) - Q(p)\beta^T\Theta\mathbf{u}(t) \\ &= y_f(t) - \beta^T\Theta\mathbf{u}_f(t) \end{aligned} \qquad (9)$$

where $\mathbf{u}(t) = \left[u(t)\ 1\right]^T$ denotes the augmented input vector. The main idea of the proposed approach is to filter both the input and the output data, thus

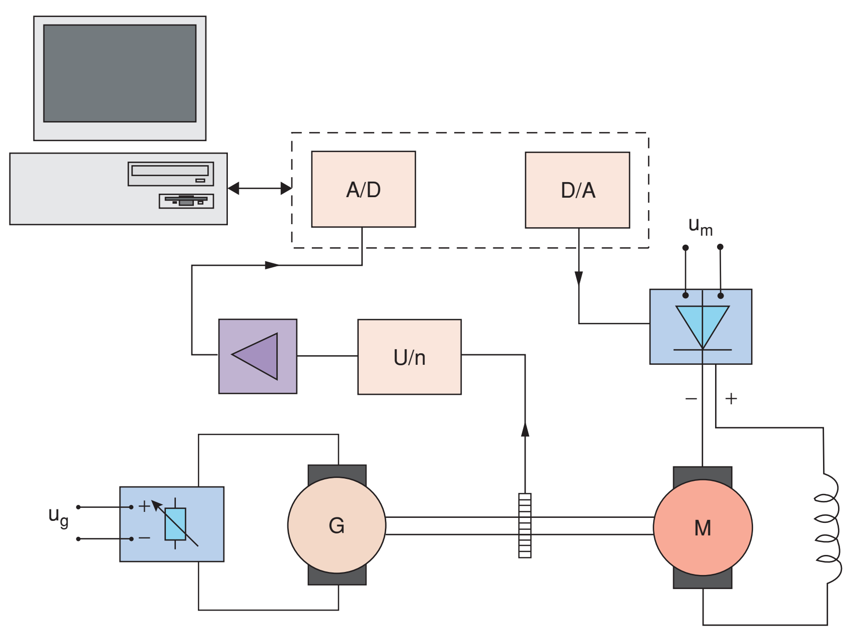

Fig. 3 Schematic representation of the motor-generator plant

obtaining a confidence band of filtered input-output data pairs, approximate the band using the optimization procedure of the INFUMO, and connect the INFUMO in parallel to the process to get online estimations of the boundary outputs. For fault detection, the decision function should consist of verifying that each measurement belongs to the corresponding confidence band. To provide quantitative information about the proximity of the measurements to the closest interval boundaries, distances were used (Fagarasan et al.). If a filtered output value $y_f(t)$ belongs to an interval $\left[\underline{y}_f(t), \overline{y}_f(t)\right]$, and if the mean interval value is denoted $\hat{y}_f(t)$, the proposed distance is defined in the following way:

$$\text{if } y_f(t) < \hat{y}_f(t),\ d(y_f) = \frac{y_f(t) - \hat{y}_f(t)}{\underline{y}_f(t) - \hat{y}_f(t)}$$

$$\text{if } y_f(t) > \hat{y}_f(t),\ d(y_f) = \frac{y_f(t) - \hat{y}_f(t)}{\overline{y}_f(t) - \hat{y}_f(t)}. \tag{10}$$

The distance in (10) is zero when the measurement is equal to $\hat{y}_f$, and approaches the value one if the measurement is close to one of the interval boundaries. A fault is signaled every time $d(y_f)$ exceeds the value one. Figure 2 gives a schematic representation of the proposed fault-detection system. The filter $Q(p)$ is represented by a block denoted LPF, and the distance is calculated in the DIST block.

Application of the INFUMO in the fault detection of a motor–generator plant

In this section, the application of the INFUMO in the robust identification and FD of a process from a class of systems with uncertain and interval-type parameters will be presented.

The electromechanical process consists of two dc motors that are mounted facing each other, as shown in Fig. 3. The driving shafts are rigidly coupled. The left motor, marked as G, is the load of the motor M when operating in generator mode. Applying a negative voltage to the generator produces mechanical torque and results in a shift of the operating conditions. The system output is the voltage obtained by a tacho generator, which is mounted to the shaft and converts the rotary speed to a dc-voltage output signal. u_m and u_g are the input voltages for the excitation and the load, respectively. The signals are connected through an analog/digital-digital/analog converter to a PC. The plant setup enables one to control the shaft speed by changing the motor's input voltage.

The process parameters are uncertain. If consecutive open-loop experiments on identical input signals are performed, the output responses will form a set of different trajectories rather than a single one. One of the reasons for such behavior is that the system performance depends on the operating temperature.

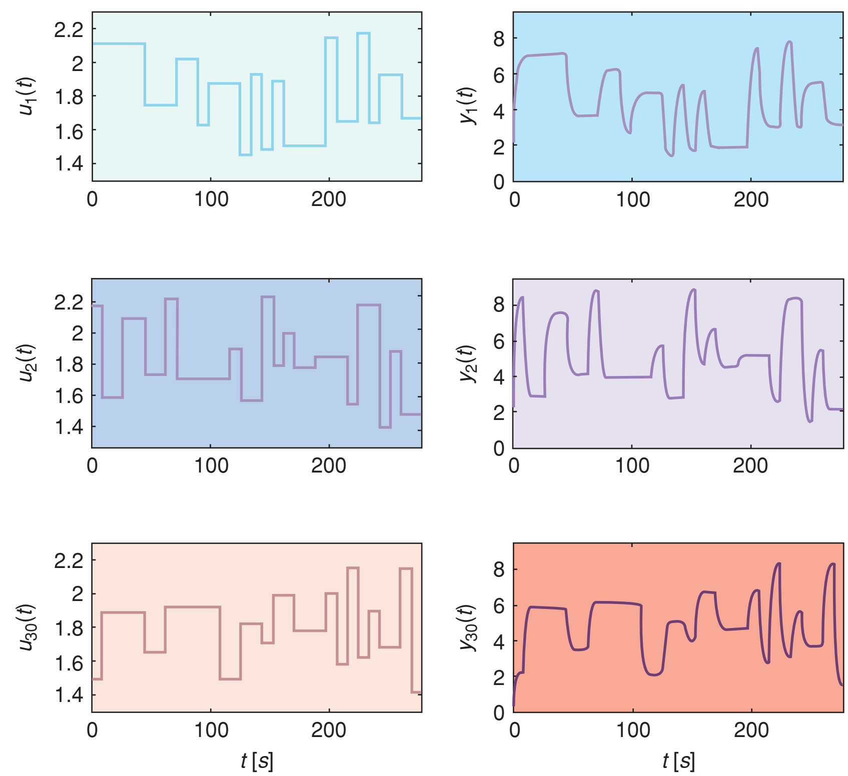

Fig. 4 Process inputs and outputs: the first, the second, and the last experiment

Experiments show that load values ranging from $u_g = 0$ V to $u_g = -0.05$ V do not shift the operating conditions substantially. Hence, the confidence load interval was defined as $[-0.05, 0]$ V. With reference to the given INFUMO identification procedure, a confidence band of input-output data must be defined. This band will represent the most significant operating range of the plant and include all unexpected deviations due to parameter uncertainties. A set of 30 experiments was carried out, i.e., five series of six identification signals at load voltages from the lowest to the highest value in 0.01 V steps. The inputs and associated output signals are shown in Fig. 4. For the sake of brevity, only the first, the second, and the last data sets are presented.

One of the major benefits of the interval fuzzy model identification, shown in Fig. 4, is that the input signals can be arbitrary. Normally, to get a confidence band of measurements, it would be necessary to that experiments with identical excitation signals are conducted. In industrial practice, however, a huge amount of various testing data is usually available when new equipment is introduced to the production process. Therefore, we feel that being able to create an FD system based on data acquired from a set of experiments on unequal signals is a benefit that could come of great use when applying the proposed method in practice.

According to (8), the input and output signals are subjected to low-pass filtering (LFP). The structure of the LPF was chosen as a simple first-order system, represented by the transfer function in (11)

$$G_f = \frac{1}{T_f s + 1}. \tag{11}$$

Optimal design of the LPF time constant was not considered in this study. The cut-off frequency must be

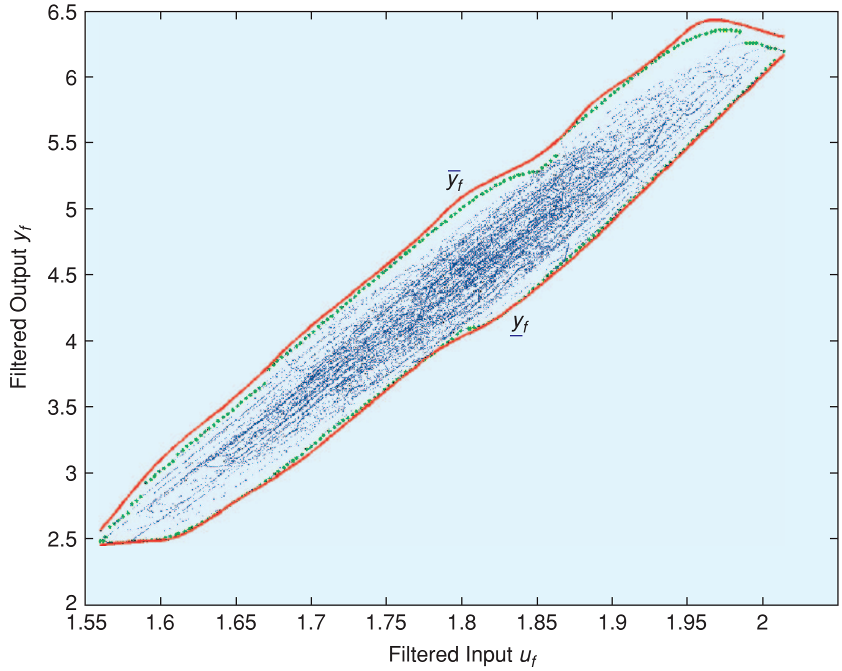

Fig. 5 Set of filtered input-output data with boundary points and boundary INFUMO functions

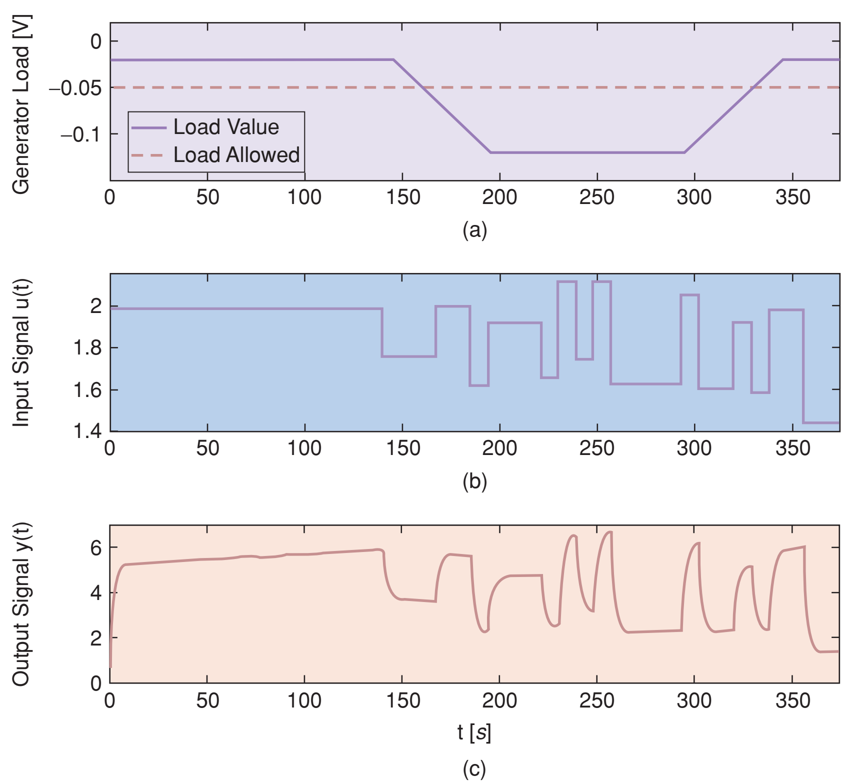

Fig. 6 Test experiment signals. Upper-load value crosses the allowed value, middle-input signal, and lower-output signal

low enough to allow only slowly changing signals to propagate through the filter. As this directly affects the choice of the time constant, a compromise has to be made to ensure that the system response is not too slow. Hence, it was chosen as $T_f = 30$ s. This way, a compact set of measurements that represents steady-state system behavior is obtained. It can be seen as a load-dependent static input-output mapping area.

The total number of points gathered from the identification experiments was 83,700. Performing the optimization on the given data set would be extremely time consuming. Therefore, data reduction is performed by determining the boundary points. First, the range of input measurements is divided into equidistant subspaces. The length of the step is chosen according to the subspace with the highest density of data. In each subspace, the extremity points are determined. The input-output data is presented in Fig. 5, and the resulting set of 302 boundary points is emphasized. These data will be used as the training data set for the INFUMO identification. A static INFUMO can be employed. This brings an additional reduction of fuzzy parameters to be optimized. The membership functions of the INFUMO antecedent variables were arranged using the Gustafson-Kessel clustering method (Babuška). According to the shape of the data area, it was sufficient to use six fuzzy subsets for the upper and lower fuzzy functions.

The parameters were optimized using the proposed INFUMO optimization algorithm in (10). The resulting boundary functions can be seen in Fig. 5. It is evident that the min-max optimization gave satisfactory results in approximating the given area.

To realize a fault-detection system, INFUMO is connected to the process in parallel, as shown in Fig. 2. In the test experiment, we used a similar input signal as in the identification experiments and simulated a fault by letting the load value cross the permitted load band. The load signal was a combination of ramps that is outside the load band in the time period $T_{ft} = 160 - 330$ s. It is presented in the upper diagram of Fig. 6. The middle and lower diagrams show the input test signal and the corresponding process output signal. Note that in the first 140 s of the experiment, the input signal was constant, so, the operating conditions were met.

The results of the test run can be seen in Fig. 7. In the diagram in Fig. 7(a), the time course of the distance, calculated by the DIST block in Fig, 2, is presented. The alarm is called when the distance crosses the threshold marked by a dashed line. The shaded area, denoted "Load boundary crossing," denotes the period when the load is not in the permitted interval. The thick solid line in the bottom of the dia-

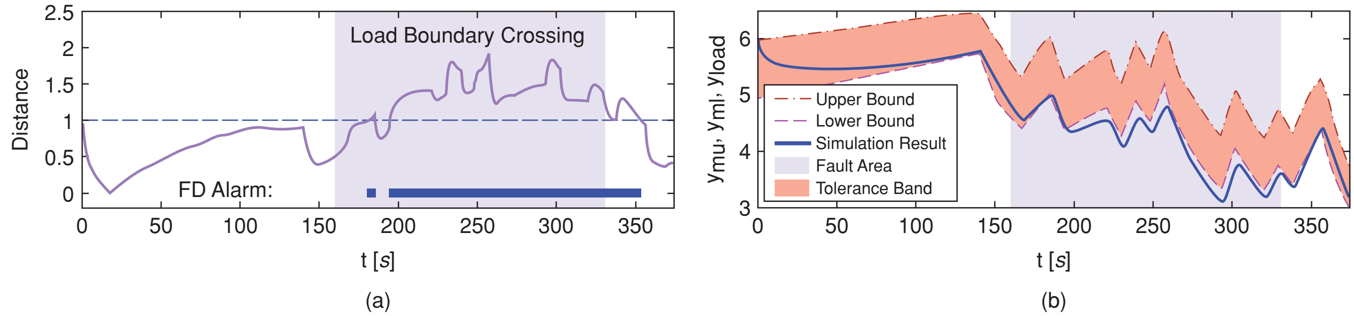

Fig. 7 Results of the fault detection system: (a) distance time course and (b) lower-test results as seen from the FD system

gram, denoted “FD alarm,” indicates the periods when the alarm is called. The lower diagram shows the comparison done by the FD system. The filtered process output is compared to the boundaries provided by the INFUMO.

It is evident that the proposed FD system successfully tracks the load crossing of the permitted band. The time needed to detect the fault was 21 s. The complete period when the alarm was called coincides with the fault period. There is a time delay in calling the alarm that depends on the width of the tolerance band and the way the process responds to the load change. On one short occasion, the alarm was not called correspondingly. This will be discussed with reference to the Fig. 7(b). It can be seen that during the second transient after the occurrence of the fault, the filtered process output crossed the boundary area for a short period of time. It can be concluded that in this case, fault prediction was not certain due to the effect of the plant’s unmodeled dynamics. Since the filter time constant can be seen as a tradeoff between robustness to the unknown process inputs and speed of the FD system, one way of reducing this uncertainty would be to find the optimal structure and parameters of the applied filter. However, the latter was not considered in this work.

Conclusions

A novel approach of INFUMO has been applied in fault detection. The INFUMO was derived using the l_∞-norm function approximation. It was shown that the INFUMO enables the confining of an arbitrary nonlinear confidence band with an upper and lower fuzzy function. It is therefore suitable for the identification of systems with uncertain parameters, as all the system responses in the given interval of uncertainty can be found in the confidence band with a probability value of one. The benefit in fault detection is to be able to directly model a family of interval-type parameter systems, which guarantees fault-tolerant action.

An application involving the load-change detection of a motor-generator pilot plant was presented. To get a confidence band of system responses, a large number of experiments was carried out, which resulted in a huge set of data. The problem of data reduction was dealt with by filtering the input and output signals using a low-pass filter. The main benefits were the possibility of using arbitrary input signals and the simplicity of the fuzzy static model that was used. The boundary points of the gathered-data set were determined by using a simple algorithm and used as a training-data set for identification by linear programming. Connecting the INFUMO to the process in parallel and employing an online calculation of the normalized distance of the filtered process output from the nearest boundary, the proposed approach was proven to be successful in detecting unwanted load changes.

Future work will concentrate on an optimization of the filter parameters, investigating the performance resulting from different choices of filter structures, and investigating possible extensions to frequency-based methods and fault-tolerant control.

Read more about it

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